**Homework 5 Stat 603**

**3.43** You take a standard deck of playing cards, and remove one card at random. You then draw a single card. Write S for the event that the card you remove is a six. Write N for the event that the card you remove is not a six. Write R for the event that the card you remove is red. Write B for the event the card you remove is black.  
(a) Write A for the event you draw a 6. What is P(A|S)?

**Solution:** P(A|S) = P (A S)/P(S), P(A|S) = Number of ways to draw a 6 from the remaining three 6s / Total number of remaining cards in the deck.

Hence 3 remaining 6s, and there are 51 remaining cards in the deck,

So, P(A|S) = (3/3) / (51/3) = 1/17.

(b) Write A for the event you draw a 6. What is P (A|N)?

**Solution:** P(A|N) = P (A N) /P(N), so P(N) = 48/52, P(A) = 4/51

P(A|N) = (4/51 \* 48/52)/48/52 = 4/51

(c) Write A for the event you draw a 6. What is P(A)?

**Solution:** P(A) = Number of ways to draw a 6 from the entire deck / Total number of cards in the entire deck. P(A) = 4/52 = 1/13

(d) Write D for the event you draw a red six. Are D and an independent? why?

**Solution:** For events to be independent, P(D|A) = P(D) and P(A|D) = P(A).

P(D) = 2/52, P(A) = 1/13, Since P(D|A) = P(A|D) = 2/52 ≠ P(D) = 2/52 ≠ P(A) = 1/13, D and A are not independent.

(e) Write D for the event you draw a red six. What is P(D)?

**Solution:** P(D) = Number of ways to draw a red six / Total number of cards in the entire deck

P(D) = 2/51.

**3.44** A student takes a multiple-choice test. Each question has N answers. If the student knows the answer to a question, the student gives the right answer, and otherwise guesses uniformly and at random. The student knows the answer to 70% of the questions. Write K for the event a student knows the answer to a question and R for the event the student answers the question correctly.  
(a) What is P(K)?

**Solution:** P(K) = 70/100 = 0.7  
(b) What is P(R|K)?

**Solution:** P(R|K) = P (R K) /P(K) = 1  
(c) What is P(K|R), as a function of N?

**Solution:** Using Bayes’ probability function,

P(K/R) = P (K and R)/P (R) = P(K) \* P(R|K) / (P(K) \* P(R|K) + P(K') P(R|K')

= 0.7\*1 / (0.7\* 1 + (1-0.7) \* 1/N)

= 0.7/ (0.7 + 0.3/N)

(d) What values of N will ensure that P(K|R) > 99%?

**Solution:**  99/100 < 0.7/ (0.7 + 0.3/N)

= 0.99 < 0.7/(1/N)

= N> 42.86 == 43

**3.**Pollution of the rivers in the United States has been a problem for many years. Consider  
the following events:  
A: the river is polluted  
B: a sample of water tested detects pollution  
C: fishing is permitted  
Assume that P (A) = 0.3, P (B|A) = 0.75, P (B|A’) = 0.20, P (C|(A ∩ B)) = 0.20  
P (C|(Ac ∩ B)) = 0.15, P (C|(A ∩ B’)) = 0.80, and P (C|(Ac ∩ B’)) = 0.90.  
(a) Find P (A ∩ B ∩ C).

**Solution:** P (A ∩ B ∩ C) = P (C|A and B) \* P (A and B)

= P (C|A and B) \* P(A)\*(B|A)

= 0.2 \* 0.75\*0.3 = 0.045

(b) Find P (B’ ∩ C).

**Solution:** Since B’∩ C = (A ∩B’∩C) U (A’∩ B’∩C)

P(B’∩C) = P (A ∩B’∩C) + P (A’∩ B’∩C)

But P (A ∩B’∩C) = P(C|A∩B’) \* P(A∩B’)

=0.8\*(1 – 0.75) \* 0.3 = 0.06

And P (A’∩ B’∩C) = 0.9 \*(1 – 0.2) \*(1 -0.3) = 0.504

P(B’∩C) = 0.06+0.504 = 0.564

(c) Find P (C).

**Solution:** C = (A ∩B’ ∩C) U (A’∩B’∩C) U (A∩B∩C) U(A’∩B∩C)

P (C) = (A ∩B’ ∩C) +(A’∩B’∩C) + (A∩B∩C) +(A’∩B∩C)

But P(A’∩B∩C) = P(C|A’∩B) \*P(A’∩B)

= 0.15 \* P(B|A’) \* P(A’)

= 0.15\* 0.20\*0.07 = 0.021

Hence P (C) = 0.06+0.504+0.045+0.021 = 0.63

(d) Find the probability that the river is polluted given that fishing is permitted and  
the sample tested did not detect pollution

**Solution:**  The required probability = P(A|C∩B’) = P(A∩B’∩C)/P(C∩B’) =0.06/0.546

= 0.106